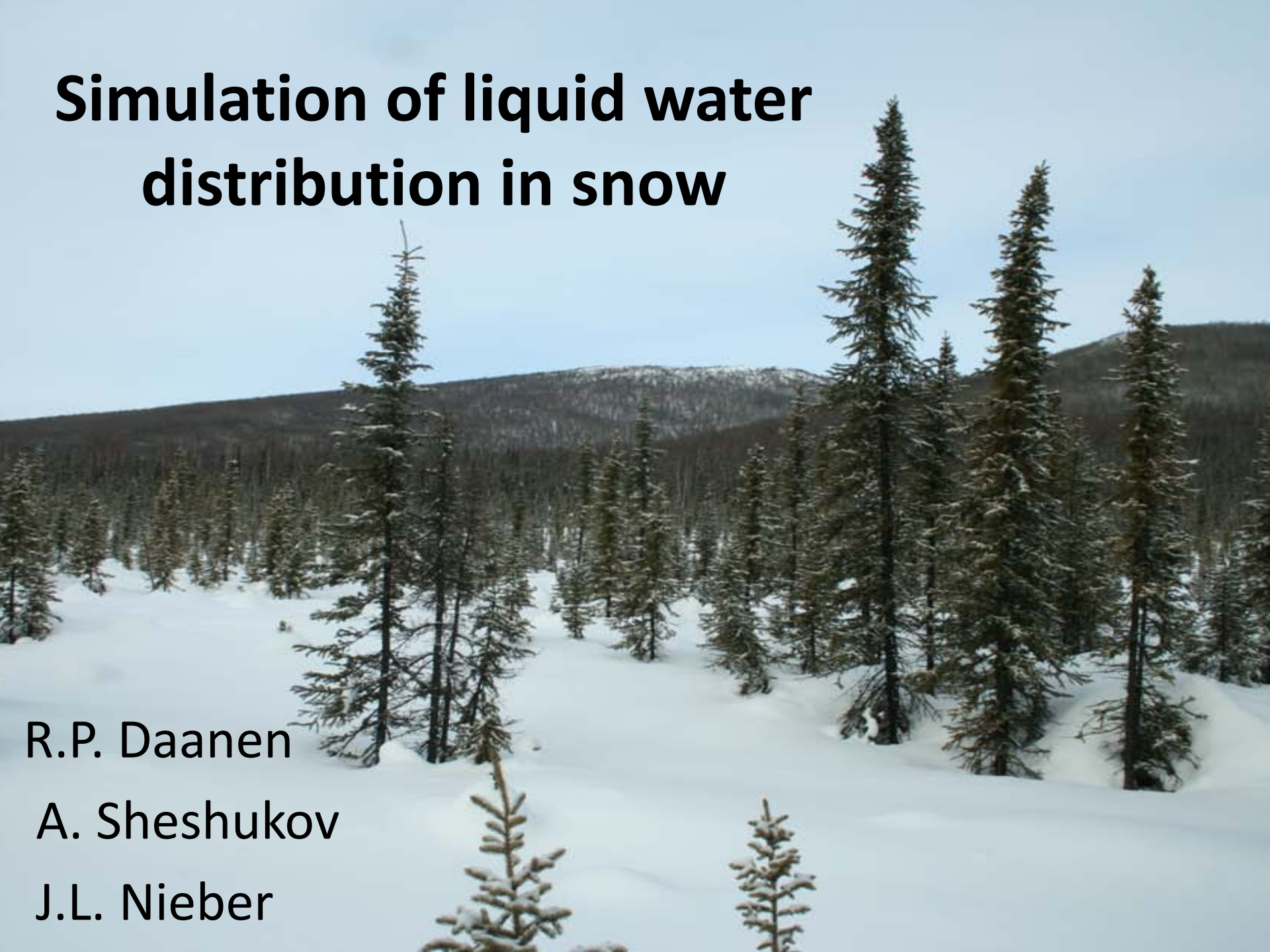
Simulation of liquid water distribution in snow



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Introduction

- Determining liquid water distribution in snow is important for
  - Snow thermal conditions
  - Melt rate of the snowpack
  - Runoff
  - Refreezing during temperature fluctuation
  - Snow crystal metamorphism
  - Formation of solid ice layers

Introduction

- Liquid water and temperature in a snowpack are closely related
  - Generally wet snow is assumed to be at zero degrees Celsius whereas dry snow is assumed to be colder than zero degrees Celsius
  - We assumed a relation between temperature and liquid water pressure using the generalized Clapeyron equation and we related this liquid water pressure to liquid water retention in the snow.

Model

- Variably saturated porous medium with a strong temperature control and phase change
  - Richards equation
  - Fourier's law with convection equation
  - Generelized Clapeyron equation
  - Van Genuchten's equation

Model Tests

- Comparison tests where presented in: Daanen, R. P., and Nieber, J. L.: Model for coupled liquid water flow and heat transport with phase change in a snowpack, Journal of Cold Regions Engineering, 23, 43-68, 2009.
- The model was tested using a self similar solution and the model tested well this paper.

Equations

Mass

$$\frac{\partial \theta_l}{\partial t} + \frac{\rho_i}{\rho_l}\frac{\partial \theta_i}{\partial t} = \nabla \cdot \left(D_l C \nabla \psi_l \right) + \boldsymbol{K}\boldsymbol{k}$$

Energy

$$\frac{\partial \left(\rho_b C_v T \right)}{\partial t} + L_f \rho_i \frac{\partial \theta_i}{\partial t} = \nabla \cdot \left(\lambda \nabla T \right) + \nabla \cdot \left(q_l T \right)$$

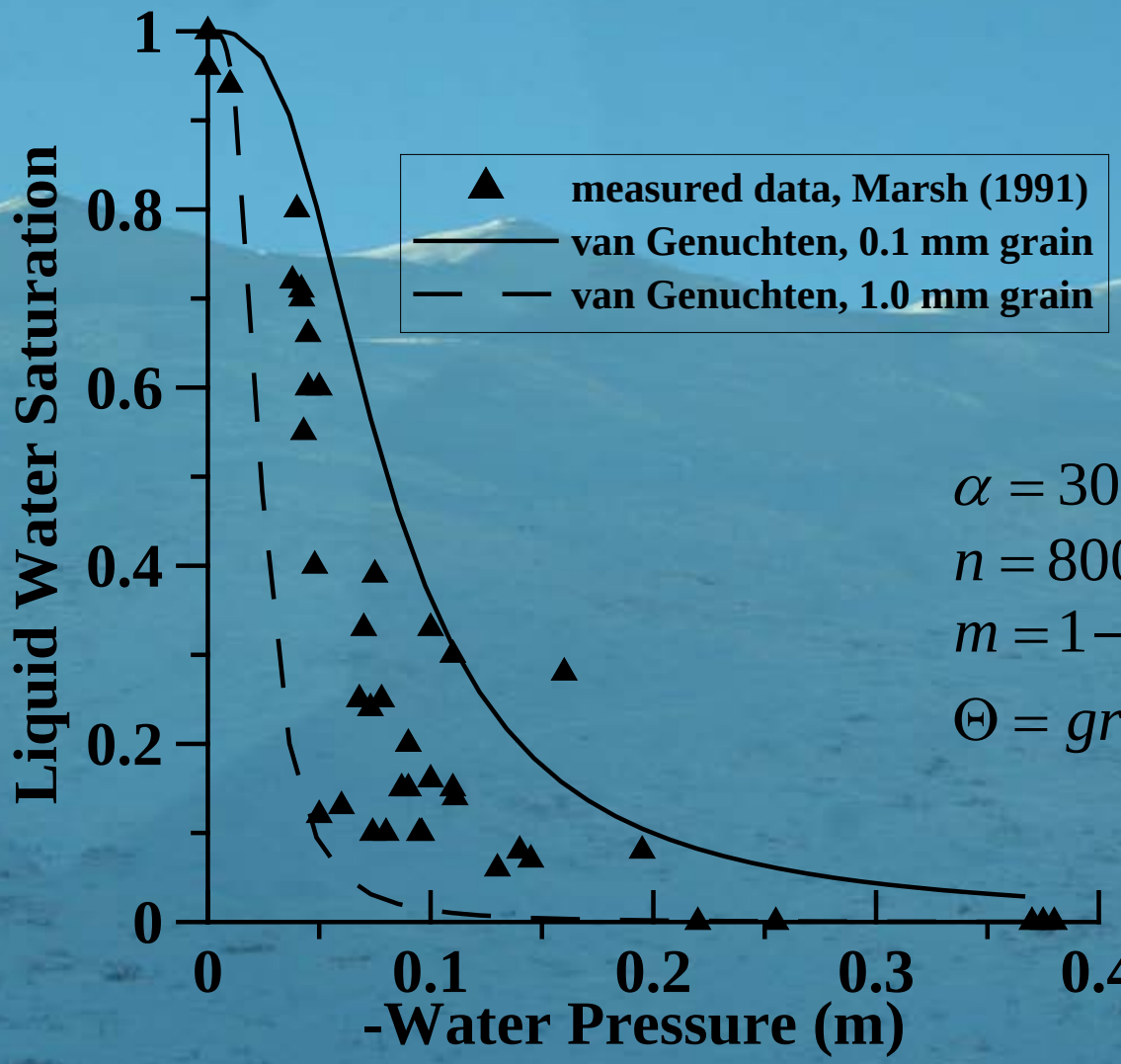
Phase change

$$\psi_l = \left(\frac{L_f}{273.15 g} \right) T + \psi_i \quad , \quad T \le 0.0\,^{o}C$$

$$\theta_l = \theta_r + \left(\theta_s - \theta_r \right) \left(\frac{1}{1 + \left(\alpha \left| \psi_l \right| \right)^{n}} \right)^{m}$$

These terms are associated with pore structure

Liquid water retention



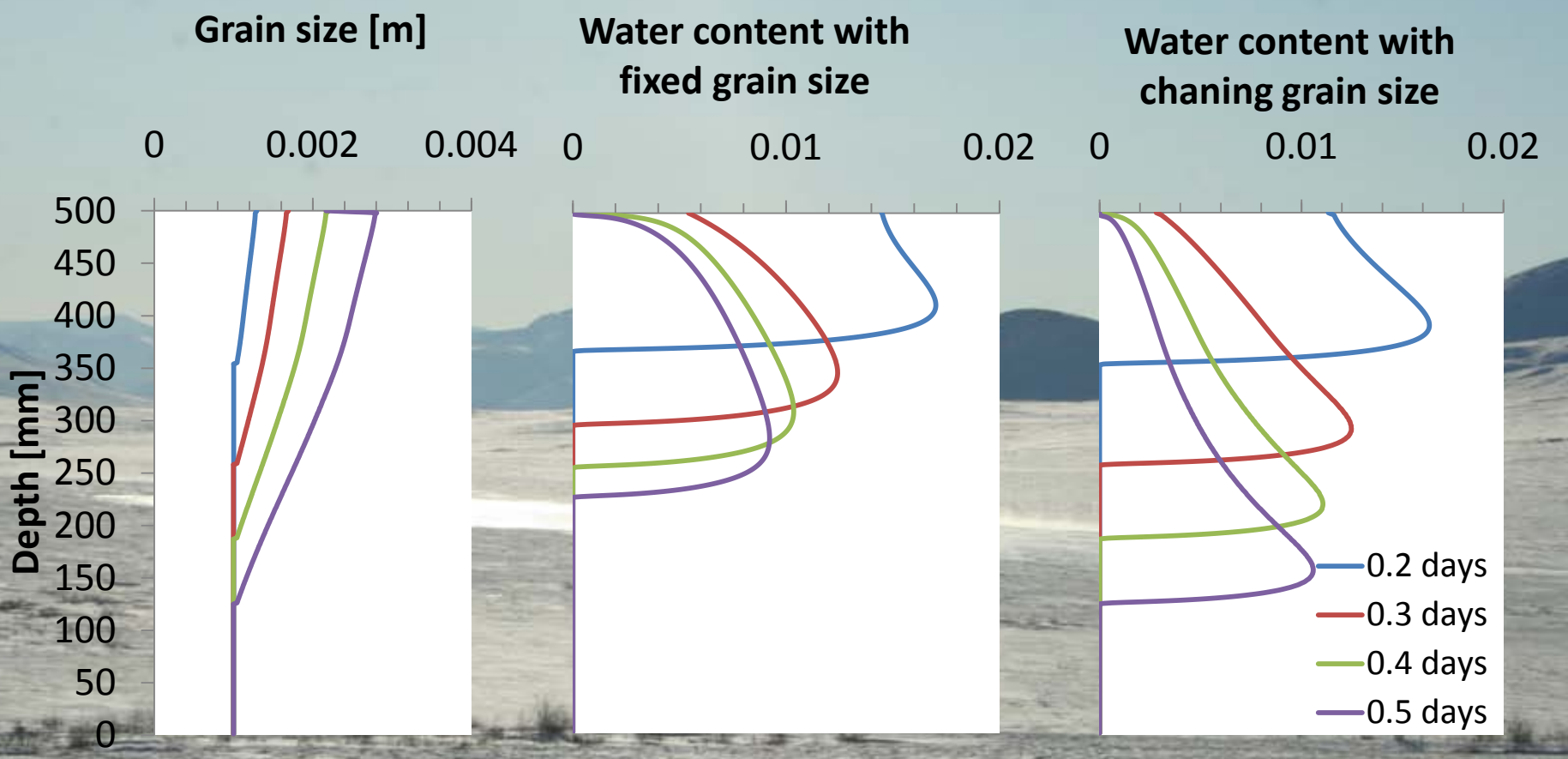
$$\alpha = 30000\,\Theta + 12$$

$$n = 800\,\Theta + 3$$

$$m = 1 - (1/n)$$

$$\Theta = grain\,size[m]$$

Results


Grain size [m]
Water content with fixed grain size
Water content with chaning grain size
0
0.002
0.004
0
0.01
0.02
0
0.01
0.02
Depth [mm]
500
450
400
350
300
250
200
150
100
50
0
0.2 days
0.3 days
0.4 days
0.5 days

Self similar solution

One dimensional horizontal flow

$$\frac{\partial \theta_l}{\partial t} + \frac{\rho_i}{\rho_l}\frac{\partial \theta_i}{\partial t} = \frac{\partial}{\partial x} D_l \frac{\partial \psi_l}{\partial x}$$

$$\frac{\partial C_v T}{\partial t} - L_f \rho_i \frac{\partial \theta_i}{\partial t} = \frac{\partial}{\partial x} \lambda \frac{\partial T}{\partial x} - C_l \frac{\partial qT}{\partial x}$$

Simplification

$$\Lambda = \frac{x}{\sqrt{t}} \qquad q = \frac{q''}{\sqrt{t}}$$

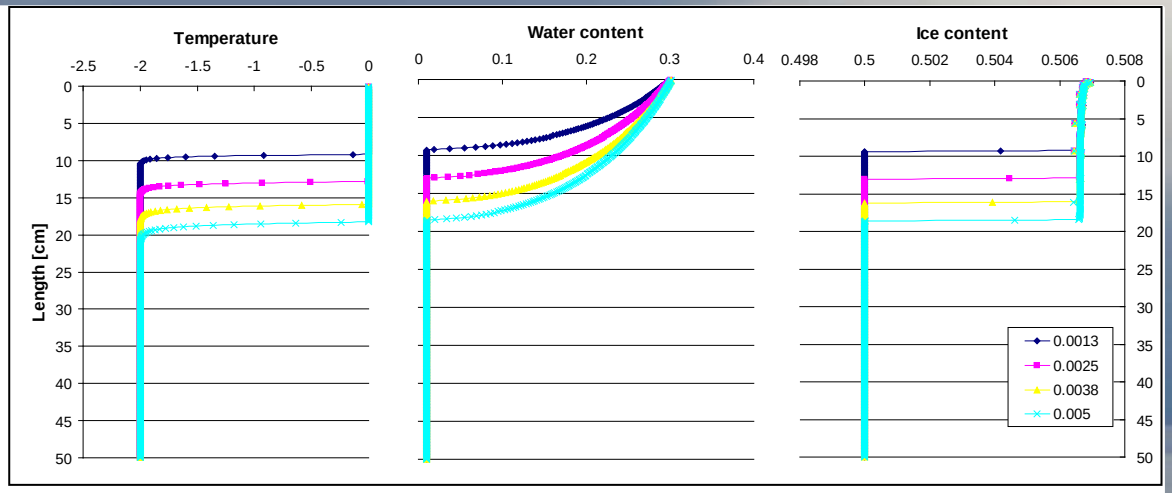
Self similar equations

$$-\frac{\Lambda}{2}\theta_l' - \frac{\Lambda}{2}\frac{\rho_i}{\rho_l}\theta_i' = (D_l \theta_l')' \qquad \left(\frac{d}{d\Lambda} = '\right)$$

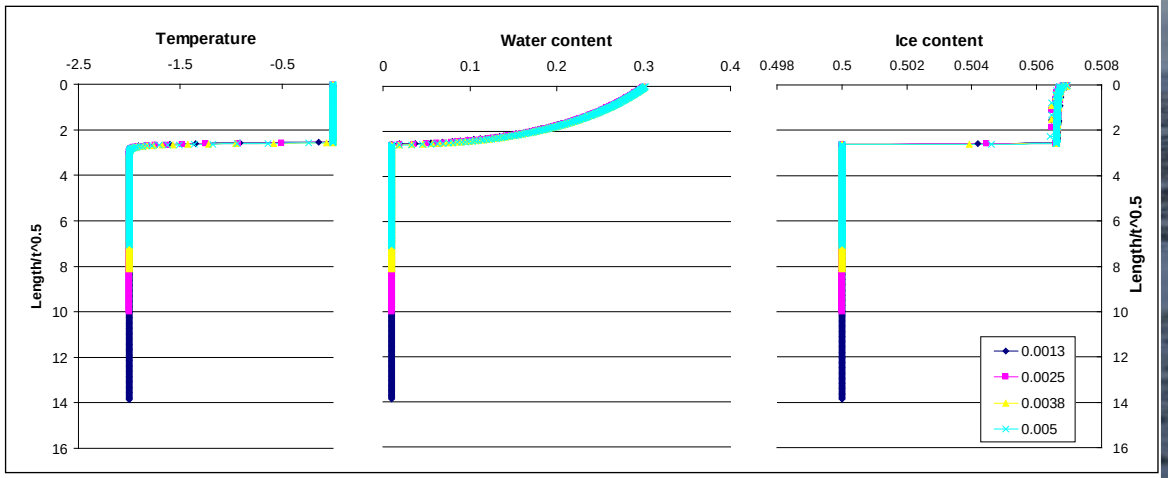
$$-\frac{\Lambda}{2} C_v T' + \frac{\Lambda}{2}\{L_{fus}\rho_i\}\theta_i' = (\lambda T')' - C_{liq} q'' T'$$

Self similar results

$$q = \frac{0.001}{\sqrt{t}}$$

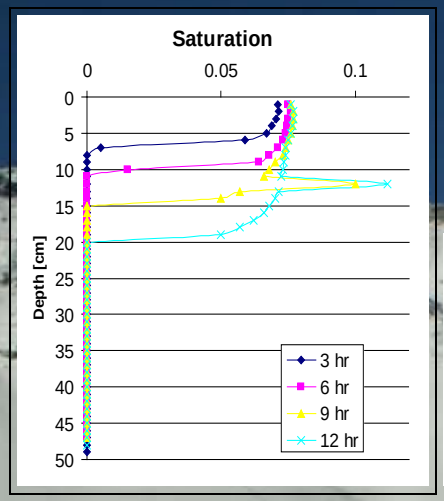


Corrected length scale

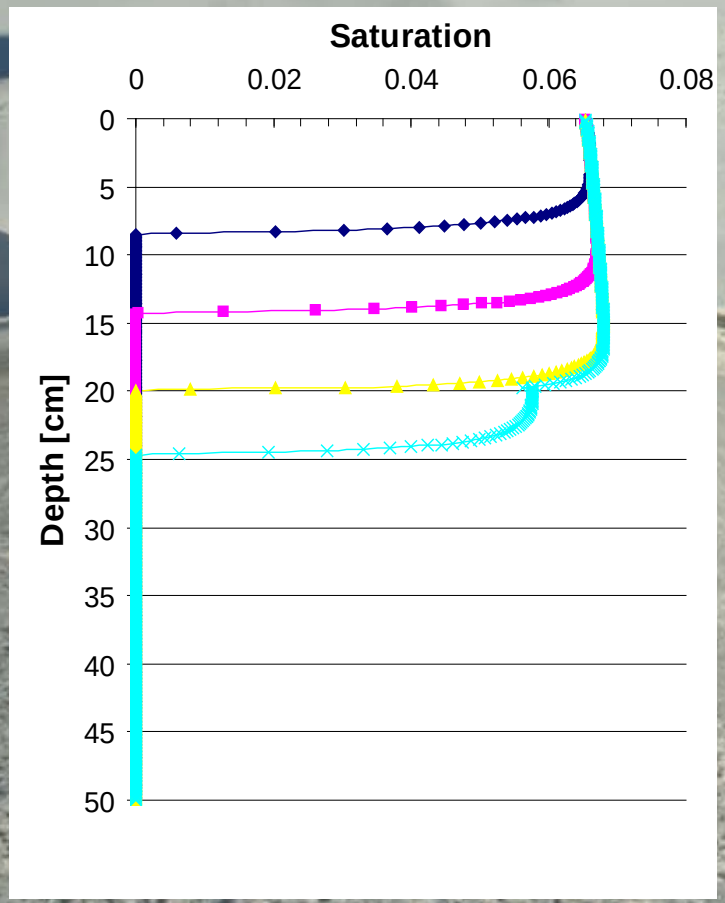


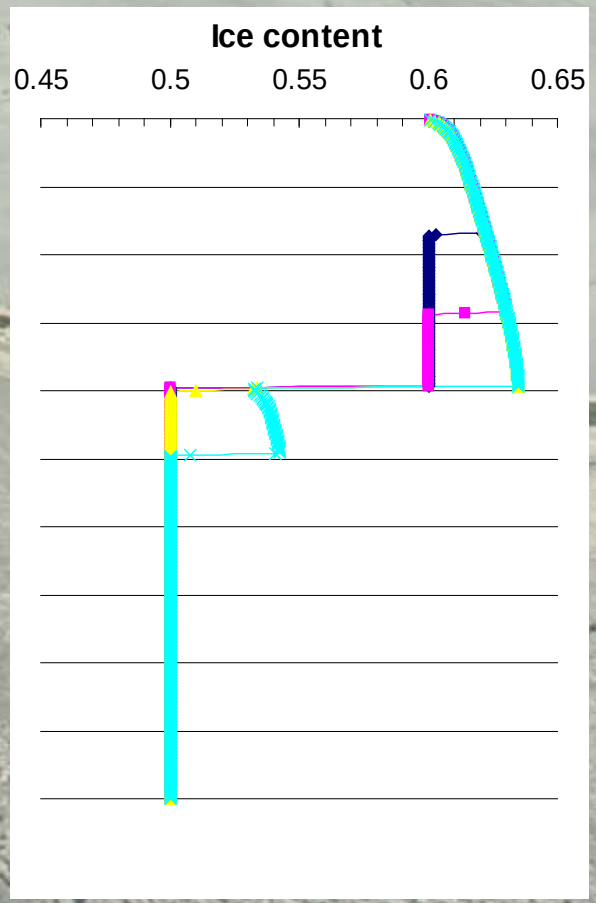
Preferential liquid water flow in snow

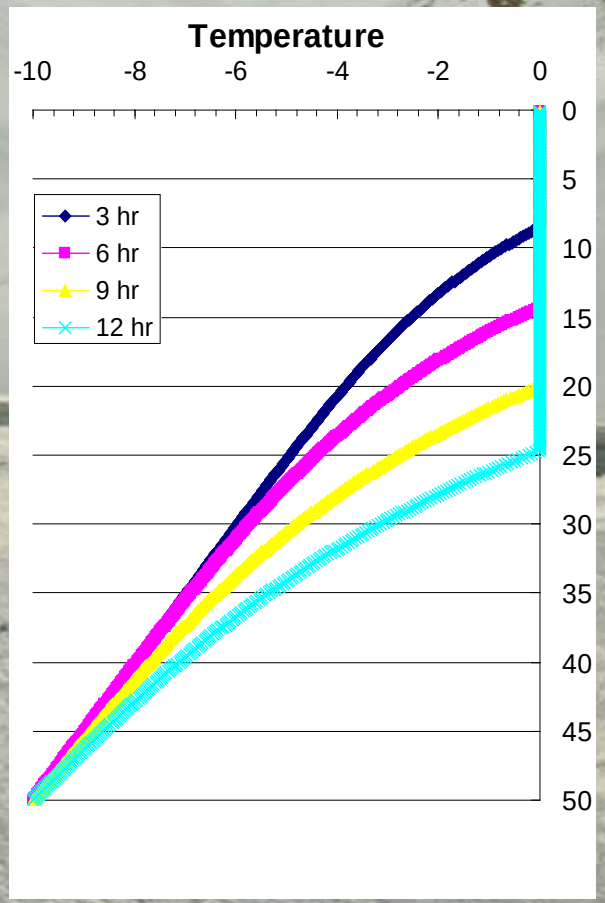
Simulation results of capillary a barrier



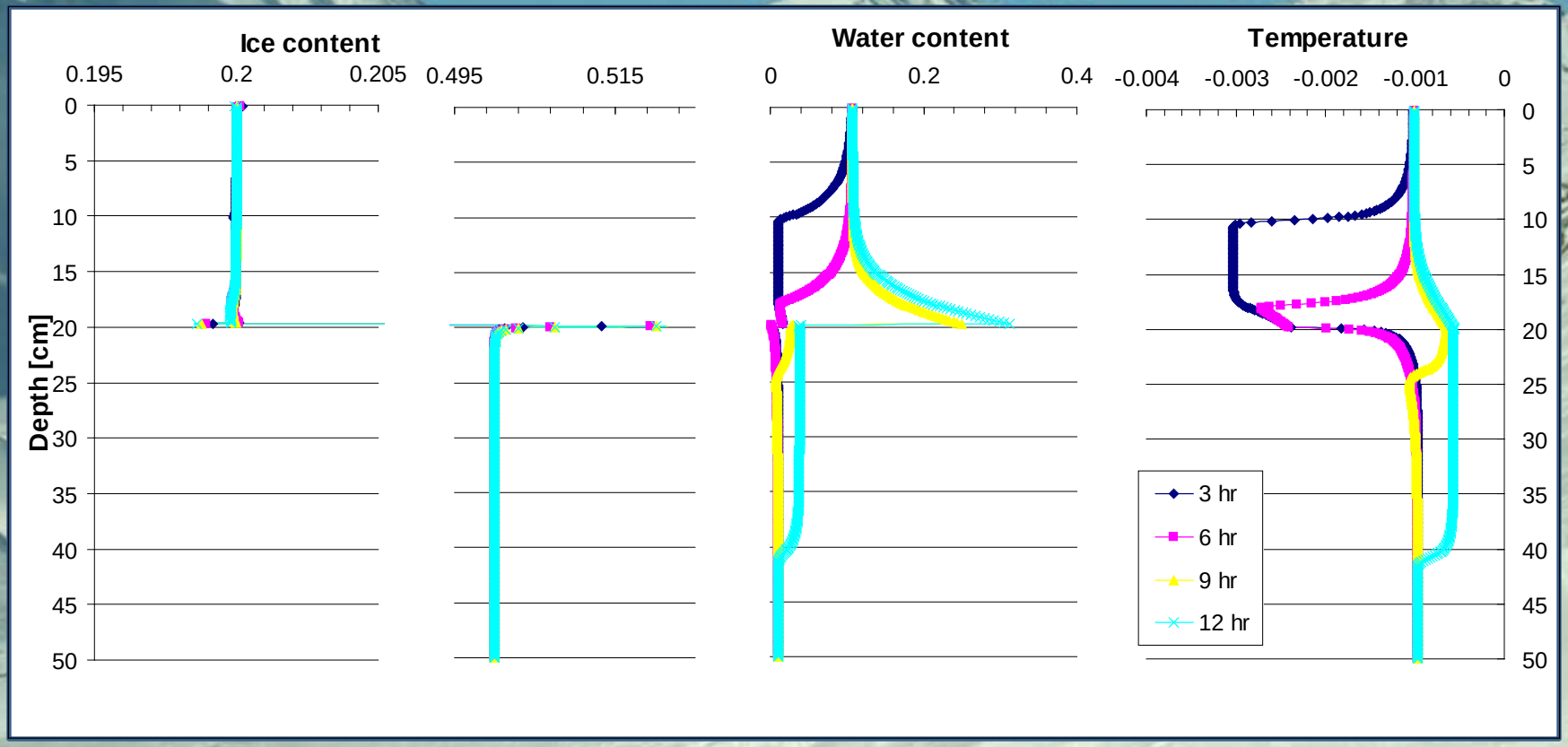
Saturation curve with abrupt drop in density data from Illangasekare et al.(1990).







Results of density and grain size barrier



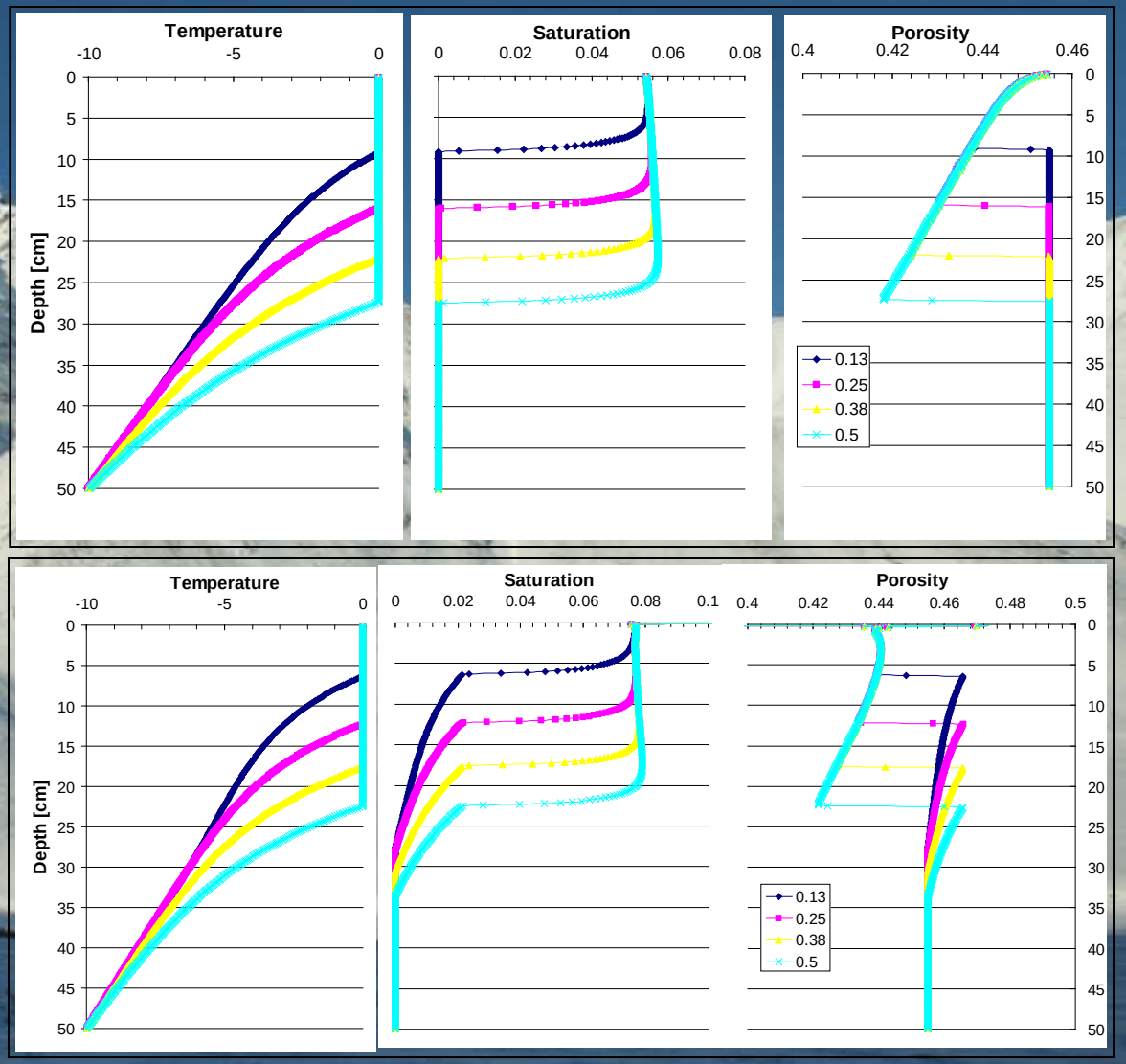
Residual water content

$$\theta_l = \theta_r + (\theta_s - \theta_r) \times Sat$$

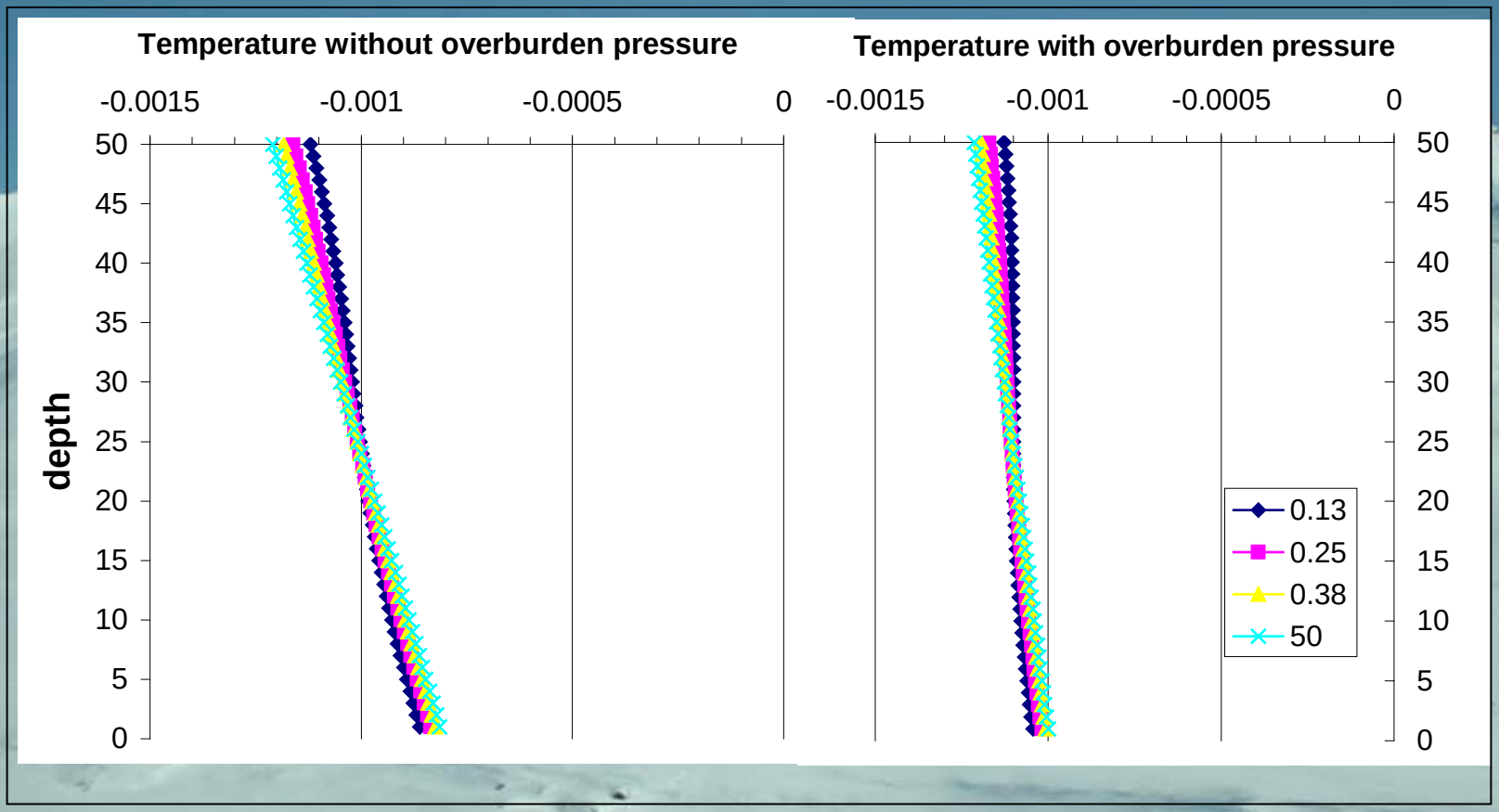
$$\theta_r = 0.0$$

$$\theta_r = 0.02$$

With a linear extension of the water retention curve to zero, below the residual water content.



Effect of gravity on the simulated temperature in a hydrostatic system



Temperature gradient forced by gravity using the assumption of zero ice pressure

Overburden pressure on the ice structure can correct for the temperature gradient